Examination Number:

Set:

Shore

Year 12

HSC Assessment Task 4

Half-Yearly Exam

May 2013

Mathematics Extension 1

General Instructions

- Reading time − 5 minutes
- Working time − 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided separately
- Answer Questions 1–10 on the Multiple Choice Answer Sheet provided
- In Questions 11–14 show relevant mathematical reasoning and/or calculations
- Start each of Questions 11–14 in a new writing booklet
- Write your examination number on the front cover of each booklet

If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A" on the front cover

Total marks – 70

Section I Pages 3–6

10 marks

- Attempt Questions 1−10
- Allow about 15 minutes for this section

Section II Pages 7–12

60 marks

- Attempt Questions 11−14
- Allow about 1 hour 45 minutes for this section

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple Choice Answer Sheet for Questions 1–10.

1 The point P divides the interval from $A(-2, 1)$ to $B(4, -5)$ internally in the ratio 4 : 3.

What is the x co-ordinate of P?

(A) $\frac{-17}{7}$

(B) $\frac{10}{7}$

(C) 10

(D) $\frac{10}{12}$

2 Simplify $2\log_x k - \log_x 3 + \log_x p$.

(A) $\log_x \frac{pk^2}{3}$

(B) $\frac{2\log_x pk}{\log_x 3}$

(C) $\frac{2\log_x k}{\log_x 3} \times \log_x p$

(D) $\frac{\log_x pk^2}{\log_x 3}$

3 What is the exact value of $\int_0^{\ln 2} \frac{e^x}{1+e^x}\,dx$?

(A) $\ln 2$

(B) 0

(C) $e^{\ln 2} - 1$

(D) $\ln 1.5$

4 What is the exact value of $\int_{-3}^{3} \frac{dx}{x^2+9}$?

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{2}{3}$

(D) $2\ln 18$

5 The curves $y = x^2$ and $y = x^3$ intersect at the point (1, 1). Which of the following is closest to the size of the acute angle in radians between these curves at (1, 1)?

(A) 0

(B) 0.14

(C) 8.13

(D) $\frac{\pi}{4}$

6 In a circle a chord of length 10 units is drawn $\sqrt{3}$ units from its centre O. What is the diameter of this circle?

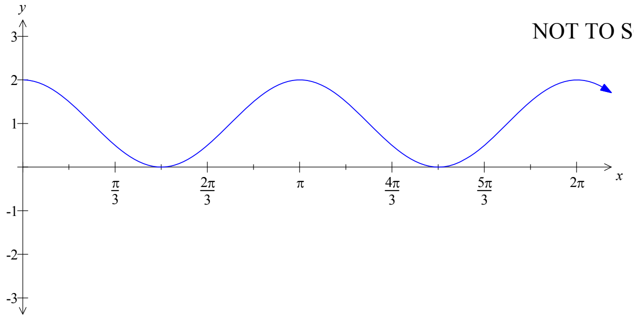
(A) $2\sqrt{103}$

(B) 28

(C) $4\sqrt{7}$

(D) $\sqrt{28}$

7 Which of the following equations best represents the graph below?



NOT TO SCALE

(A) $y = \sin 2x + 1$

(B) $y = 1 + 2\cos x$

(C) $y = 2\cos 2x$

(D) $y = 1 + \cos 2x$

8 Which one of these functions has an inverse relation that is not a function?

(A) $y = x^3$

(B) $y = \ln x$

(C) $y = \sqrt{x}$

(D) $y = |x|$

9 Given $\frac{d}{dx}(x\log_e x) = 1 + \log_e x$. Find $\int \frac{1+\log_e x}{x\log_e x}\,dx$.

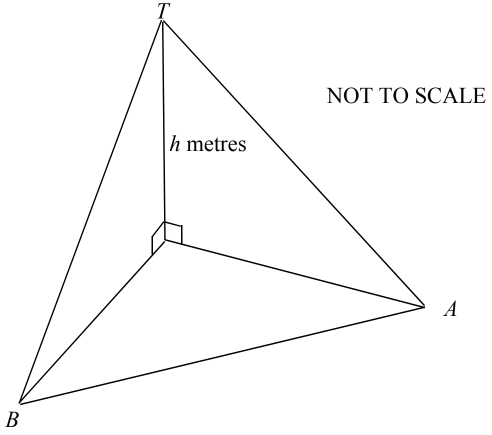
(A) $\frac{1}{x} + c$

(B) $\log_e(1 + \log_e x) + c$

(C) $\log_e(x\log_e x) + c$

(D) $(x\log_e x) + c$

10 A tower *T, h* metres high can be seen from a point *A* due East of the tower and point *B* due south of the tower. If the distance between *A* and *B* is 60 metres and the angles of elevation of *T* from *A* and *B* are $14°$ and $17°$ respectively, find the height of the tower.



(A) 11.6

(B) 134.4

(C) 12.2

(D) 5.5

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Start each of Questions 11–14 in a new writing booklet.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Find the value of k if $x + 3$ is a factor of $P(x) = x^3 - 3kx + 1$. **1**

(b) Solve the inequality $\frac{3x}{2-x} \geq -1$. **3**

(c) Evaluate $\lim_{x\to 0}\frac{\sin 4x}{3x}$. **2**

(d) Find $\frac{d}{dx}\left(x\tan^{-1}3x\right)$. **2**

(e) Find $\int \frac{1}{\sqrt{25 - 9x^2}}\,dx$. **2**

(f) Find the exact value of $\sin\left(\cos^{-1}\frac{2}{7}\right)$. **2**

(g) If α, β, γ are the roots of $2x^3 - 5x^2 + 3x - 1 = 0$, find the value of $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$. **3**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Consider the function $y = 2\cos^{-1}\frac{x}{3}$.

(i) State the domain and range. **2**

(ii) Sketch the curve. **1**

(iii) Find the area between the curve and the *x* and *y* axes in the first quadrant. **3**

(b) Use the substitution $u = x^2 + 1$ to evaluate $\int_0^1 x^3(x^2+1)^3\,dx$. **3**

(c) Consider the function $f(x) = \frac{x}{x+3}$.

(i) Show that $f'(x) > 0$ for all *x* in the domain. **1**

(ii) State the equation of the horizontal asymptote of $y = f(x)$. **1**

(iii) Without using further calculus, sketch the graph of $y = f(x)$. **1**

(iv) Explain why $y = f(x)$ has an inverse function $y = f^{-1}(x)$. **1**

(v) Find the inverse function $y = f^{-1}(x)$. **1**

(vi) State the domain of $y = f^{-1}(x)$. **1**

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) Solve $2\sin^2 x = \sin x$ for $0 \le x \le 2\pi$. **2**

(b) In the diagram, *AB* is a diameter of the circle, centre at *O*. The radius *OD* is extended to meet the tangent *BC* at *C*. The length of the diameter is 24 cm and $\angle AOC = 150°$.

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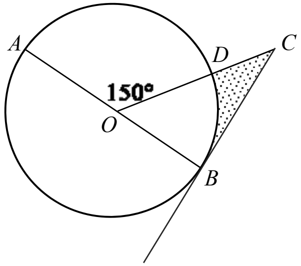


(i) Find the size of $\angle COB$ in radians. **1**

(ii) Find the exact value of the shaded area which is contained by the intervals *BC* and *CD* and the arc *BD*. **2**

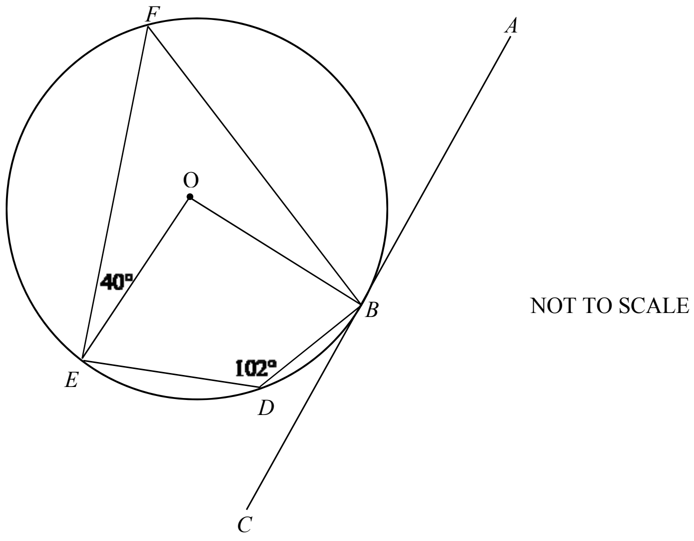
(c) Consider the function $f(x) = \frac{\ln x}{x}$. **3**

Find the co-ordinates of the stationary point on the curve $y = f(x)$ and determine its nature.

(d) Use mathematical induction to prove that for all integers $n \ge 1$, $3^{2n-1} + 5$ is divisible by 8. **3**

Question 13 continued on the next page

(e)



The points *B, D, E, F* lie on the circle with centre *O*. *AC* is a tangent to the circle touching at the point *B*.

(i) Show that $\angle BOE = 156°$. **2**

(ii) Find the size of the angle *FBA*. **2**

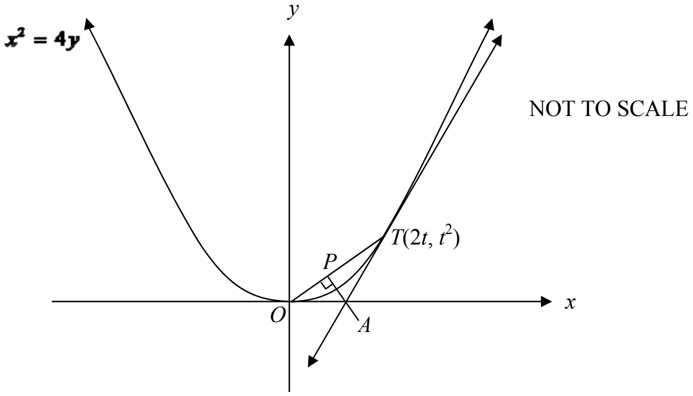
Question 14 on the next page

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) Find the exact value of the volume of the solid of revolution formed when the region bounded by the curve $y = 3\sin x$, the x-axis and the lines $x = 0$ and $x = \dfrac{\pi}{2}$ is rotated about the x-axis. **3**

(b) Prove that $\dfrac{\sin 2x}{1 + \cos 2x} = \tan x$. **2**

(c)



The tangent at $T(2t, t^2)$, $t \neq 0$ on the parabola $x^2 = 4y$ meets the x- axis at A. $P(x, y)$ is the foot of the perpendicular from A to OT, where O is the origin. The equation of the tangent at T is $y = tx - t^2$.

(i) Find the co-ordinates of the point *A*. **1**

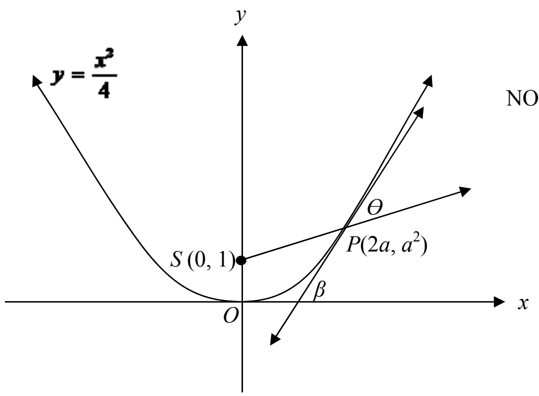
(ii) Show that the equation of *AP* is $y = -\dfrac{2}{t}(x - t)$. **2**

(iii) Show that the equation of *OT* is $t = \dfrac{2y}{x}$. **1**

(iv) Hence, or otherwise, prove that the locus of $P(x, y)$ lies on a circle with centre $(0, 1)$ and give its radius. **2**

Question 14 continued on the next page

(d)



NOT TO SCALE

Let $P\ (2a,\ a^2)$ be a point on the parabola $y = \frac{x^2}{4}$, and let S be the point (0,1). The tangent to the parabola at P makes an angle of β with the x axis. The angle between SP and the tangent is Θ. Assume $a > 0$, as indicated.

(i) Show that $\tan\beta = a$. **1**

(ii) Show that the gradient of SP is $\frac{1}{2}\left(a - \frac{1}{a}\right)$. **1**

(iii) Show that $\tan\theta = \frac{1}{a}$. **2**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x\ , \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

YEAR 12 EXTENSION 1 MATHS HALF-YEARLY 2013

1	B
2	A
3	D
4	B
5	B
6	C
7	D
8	D
9	C
10	A

1. A $(-2,1)$ B$(4,-5)$

4 : 3

$$x = \frac{4\times 4 + 3\times -2}{7}$$

$$= \frac{10}{7} \quad (B)$$

2. $\log k^2 - \log 3 + \log p$

$$= \log\left(\frac{pk^2}{3}\right) \quad (A)$$

3. $$\int_0^{\ln 2} \frac{e^x}{1+e^x}\,dx = \Big[\ln(1+e^x)\Big]_0^{\ln 2}$$

$$= \ln(1+e^{\ln 2}) - \ln(1+e^0)$$

$$= \ln 3 - \ln 2$$

$$= \ln \frac{3}{2}$$

$$= \ln 1{\cdot}5 \quad (D)$$

$(e^{\ln 2} = 2)$

4 $$\int_{-3}^{3} \frac{dx}{x^2+9} = \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_{-3}^{3}$$

$$= \frac{1}{3}\left[\tan^{-1}\frac{3}{3} - \tan^{-1}\frac{(-3)}{3}\right]$$

$$= \frac{1}{3}\left[\tan^{-1}1 - \tan^{-1}(-1)\right]$$

$$= \frac{1}{3}\left[\frac{\pi}{4} + \frac{\pi}{4}\right]$$

$$= \frac{\pi}{6} \quad (B)$$

5. $y = x^2$ $y = x^3$

$y' = 2x$ $y' = 3x^2$

At $x=1$ At $x=1$

$y' = 2$ $y' = 3$

$m_1 = 2$ $m_2 = 3$

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$$

$$= \left|\frac{2-3}{1+6}\right|$$

$$= \frac{1}{7}$$

$\theta = 0{\cdot}14$ (B)

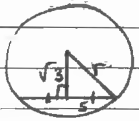
6 $r^2 = (\sqrt{3})^2 + 5^2$

$= 28$

$r = \sqrt{28}$

$= 2\sqrt{7}$

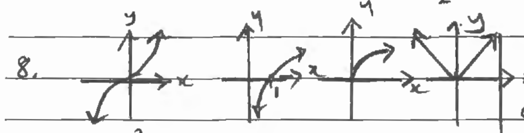
$\therefore d = 4\sqrt{7}$ (C)



7. $a = 1$ $P = \frac{2\pi}{b}$

$b = \frac{2\pi}{\pi} = 2$

$y = 1 + \cos 2x$ [D]

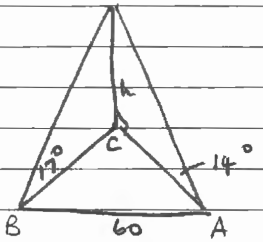
8. 

$y = x^3$ $y = \ln x$ $y = \sqrt{x}$ $y = |x|$

$\therefore$ D.

9. $$\int \frac{1+\ln x}{x\ln x}\,dx \qquad \left[\int \frac{f'(x)}{f(x)}\right]$$

$= \ln(x\ln x) + c$ [C]

10. 

$$\tan 14° = \frac{h}{AC}$$

$$AC = \frac{h}{\tan 14} \qquad BC = \frac{h}{\tan 17}.$$

In $\triangle ABC$ Pythagoras' theorem

$$60^2 = \frac{h^2}{\tan^2 14} + \frac{h^2}{\tan^2 17}$$

$$60^2 \div \left(\frac{1}{\tan^2 14} + \frac{1}{\tan^2 17}\right) = h^2$$

$$\sqrt{60^2 \div 26{\cdot}78} = h$$

$h = 11{\cdot}6$ [A]

Question 11

a) $P(x) = x^3 - 3kx + 1$

$P(-3) = (-3)^3 - 3(-3)k + 1$

$0 = -27 + 9k + 1$

$\frac{26}{9} = k$

$k = 2\frac{8}{9}$ [1]

b) $\frac{3x}{2-x} \geq -1$ [3]

1. $x \neq 2$

2 Solve $\frac{3x}{2-x} = -1$

$3x = -2 + x$

$2x = -2$

$x = -1$

3. 

$-1 \leq x < 2$

c) $$\lim_{x\to 0}\frac{\sin 4x}{3x} = \lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{4}{3}$$

$$= 1 \times \frac{4}{3}$$

$$= \frac{4}{3} \quad [2]$$

d) $$\frac{d}{dx}(x\tan^{-1}3x) = x \times \frac{3}{1+9x^2} + \tan^{-1}3x$$

$$= \frac{3x}{1+9x^2} + \tan^{-1}3x \quad [2]$$

$u = x$ $v = \tan^{-1}3x$

$u' = 1$ $v' = \frac{3}{1+9x^2}$

e) $\int \frac{1}{\sqrt{25-9x^2}}\,dx = \int \frac{1}{\sqrt{9(\frac{25}{9}-x^2)}}\,dx$

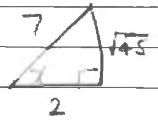
$= \frac{1}{3}\sin^{-1}\frac{x}{5/3} + c$

$= \frac{1}{3}\sin^{-1}\frac{3x}{5} + c$ [2]

f) $\sin\left(\cos^{-1}\frac{2}{7}\right) = \sin x$

let $x = \cos^{-1}\frac{2}{7}$

$\cos x = \frac{2}{7}$



$\therefore \sin x = \frac{\sqrt{45}}{7}$

$= \frac{3\sqrt{5}}{7}$ [2]

g) $\alpha^2\beta\gamma + \alpha\beta^2\gamma + \alpha\beta\gamma^2$ [3]

$= \alpha\beta\gamma(\alpha+\beta+\gamma)$

$= -\frac{d}{a} \times -\frac{b}{a}$

For $2x^3 - 5x^2 + 3x - 1 = 0$

$\alpha+\beta+\gamma = \frac{5}{2}$ $\alpha\beta\gamma = \frac{1}{2}$

$\therefore \alpha\beta\gamma(\alpha+\beta+\gamma) = \frac{5}{2} \times \frac{1}{2}$

$= \frac{5}{4}$

Question 12

(a) $-1 \leq \frac{x}{3} \leq 1$ [2]

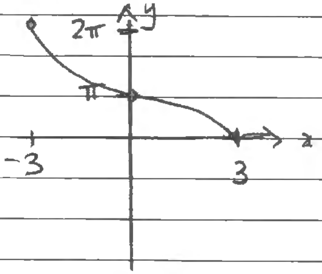
Domain:

$-3 \leq x \leq 3$

Range: $0 \leq \frac{y}{2} \leq \pi$

$0 \leq y \leq 2\pi$

(ii)



[1]

(iii) $A = \int 2\cos^{-1}\frac{x}{3}\,dx$ [3]

$y = 2\cos^{-1}\frac{x}{3}$

$\frac{y}{2} = \cos^{-1}\frac{x}{3}$

$\cos\frac{y}{2} = \frac{x}{3}$

$3\cos\frac{y}{2} = x$

$A = \int_0^{\pi} 3\cos\frac{y}{2}\,dy$

$= \left[\frac{3\sin\frac{y}{2}}{\frac{1}{2}}\right]_0^{\pi}$

$= 6\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= 6(1-0)$

$= 6\,u^2$

[3]

(b) $\int_0^1 x^3(x^2+1)^3\,dx = \int_0^1 x^2(x^2+1)^3\,x\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\frac{du}{2} = x\,dx$

$x=1$ $u=2$

$x=0$ $u=1$

$x^2 = u-1$

$= \int_1^2 (u-1)u^3\,\frac{du}{2}$

$= \frac{1}{2}\int_1^2 (u^4 - u^3)\,du$

$= \frac{1}{2}\left[\frac{u^5}{5} - \frac{u^4}{4}\right]_1^2$

$= \frac{1}{2}\left[\left(\frac{2^5}{5} - \frac{2^4}{4}\right) - \left(\frac{1}{5} - \frac{1}{4}\right)\right]$

$= \frac{49}{40}$

$= 1\frac{9}{40}$

(c) $f(x) = \frac{x}{x+3}$ $x \neq -3$

(i)

$f'(x) = \frac{(x+3)\times 1 - x \times 1}{(x+3)^2}$

$= \frac{x+3-x}{(x+3)^2}$

$= \frac{3}{(x+3)^2}$

Since $(x+3)^2 > 0$ for all x

$f'(x) > 0$ for all x [1]

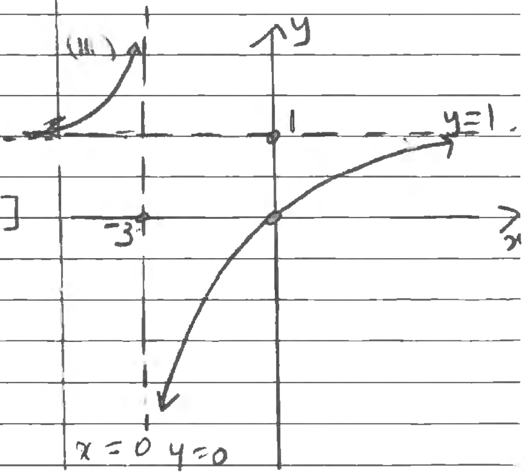
(ii) horizontal asymptote

$y = \frac{\frac{x}{x}}{\frac{x}{x}+\frac{3}{x}} = \frac{1}{1+\frac{3}{x}}$

As $x \to \infty$ $y \to 1$

$\therefore y = 1$ [1]

(iii)



$x = 0$ $y = 0$

iv) For every y value in the range of the original function there is only one x value.
[one to one mapping] [1]

(v) $y = \frac{x}{x+3}$

$x = \frac{y}{y+3}$

$xy + 3x = y$

$3x = y - xy$

$3x = y(1-x)$

$\frac{3x}{1-x} = y$

$f^{-1}(x) = \frac{3x}{1-x}$ [1]

vi) Domain: all real x except $x = 1$ [1]

Question 13.

(a) [2]

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0$ or $2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = 0, \pi, 2\pi$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\therefore x = 0, \pi, 2\pi, \frac{\pi}{6}, \frac{5\pi}{6}$

(b)(i) $\angle COB = \frac{\pi}{6}$ [1]

(ii) Shaded Area $= A_\Delta - A_{sector}$

$= \frac{1}{2} \times 12 \times 4\sqrt{3} - \frac{1}{2} \times 12^2 \times \frac{\pi}{6}$

$= 24\sqrt{3} - 12\pi$

$= 12(2\sqrt{3} - \pi)\, v^2$ [2]

$\tan \frac{\pi}{6} = \frac{h}{12}$

$h = 12 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= 4\sqrt{3}$

[3]

(c) $f(x) = \frac{\ln x}{x}$

$u = \ln x$
$u' = \frac{1}{x}$
$v = x$
$v' = 1$

$f'(x) = \frac{vu' - uv'}{v^2}$

$= \frac{x \times \frac{1}{x} - \ln x \cdot 1}{x^2}$

$= \frac{1 - \ln x}{x^2}$

Stat. pts when $f'(x) = 0$

$\frac{1 - \ln x}{x^2} = 0$ $x \neq 0$

$1 - \ln x = 0$

$1 = \ln x$

$e^1 = x$

When $x = e$ $f(e) = \frac{1}{e}$

$\therefore$ Stat pt at $(e, \frac{1}{e})$

x	2	e	3
$f'(x)$	+	0	−

$\therefore$ Max turning pt at $(e, \frac{1}{e})$

(d) [3]

Step 1 Prove true for $n = 1$

$3^{2-1} + 5 = 8$

which is divisible by 8.

Step 2: Assume true for $n = k$

$3^{2k-1} + 5 = 8M$ where M is an integer

$\therefore 3^{2k-1} = 8M - 5$.

Now prove true for $n = k+1$

$3^{2(k+1)-1} + 5 = 3^{2k+1} + 5$

$= 3^{2k-1} \cdot 3^2 + 5$

(but $3^{2k-1} = 8M - 5$)

$= 9(8M - 5) + 5$

$= 72M - 45 + 5$

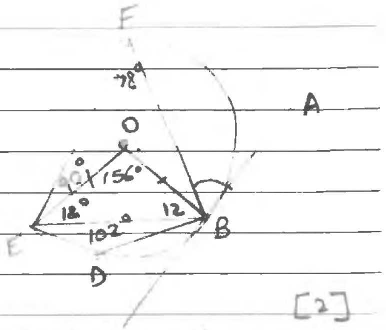
$= 72M - 40$

$= 8(9M - 5)$,

which is divisible by 8

Step 3. Since true for $n = 1$, it is true for $n = 1+1 = 2$, $n = 3$ & so on for all $n \geq 1$

(e)



[2]

(i) $\angle EFB = 78^\circ$ (opposite ∠s in cyclic quad supplementary

$\angle BOE = 156^\circ$ (angle at centre is twice ∠ at circumference standing on the same arc)

(ii) In $\triangle OEB$ [2]

$\angle OEB = (180 - 156) \div 2$

$= 12^\circ$ (base ∠s of isosceles $\triangle$)

$\therefore \angle FBA = \angle FEB = 52^\circ$

(angle between chord and tangent = angle in alternate segment)

Question 14

a)

$V = \pi \int y^2 \, dx$ [3]

$= \pi \int 9\sin^2 x \, dx$

$= 9\pi \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx$

$= \frac{9\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}}$

$= \frac{9\pi}{2}\left[\left(\frac{\pi}{2} - \frac{1}{2}\sin\pi\right) - (0-0)\right]$

$= \frac{9\pi}{2}\left[\frac{\pi}{2} - 0\right]$

$= \frac{9\pi^2}{4} u^3.$

b) LHS $= \frac{2\sin x \cos x}{1 + \cos^2 x - \sin^2 x}$ [2]

$= \frac{2\sin x \cos x}{(1-\sin^2 x) + \cos^2 x}$

$= \frac{2\sin x \cos x}{\cos^2 x + \cos^2 x}$

$= \frac{\cancel{2}\sin x \cancel{\cos x}}{\cancel{2}\cancel{\cos^2 x}\ \cos x}$

$= \frac{\sin x}{\cos x}$

$= \tan x$

$=$ RHS

(c)(i) $y = tx - t^2$

let $y=0$ $\quad t^2 = tx$

$t = x$

$\therefore \underline{A(t, 0)}$ [1]

(ii) $m_{OT} = \frac{t^2 - 0}{2t - 0}$ [2]

$= \frac{t}{2}$

$\therefore m_{AP} = -\frac{2}{t}$ (as OT ⊥ PA)

$y - 0 = -\frac{2}{t}(x - t)$

$\therefore y = -\frac{2}{t}(x - t)$ (i)

(iii) Eqn OT [1]

$y - t^2 = \frac{t}{2}(x - 2t)$

$2y - 2t^2 = tx - 2t^3$

$2y = tx$

$\frac{2y}{x} = t$

(iv) Pt of intersection $P(x, y)$ [2]

sub $t = \frac{2y}{x}$ into (i)

$y = \frac{-2}{\frac{2y}{x}}\left(x - \frac{2y}{x}\right)$

$= -\frac{\cancel{2}x}{\cancel{2}y}\left(\frac{x^2 - 2y}{x}\right)$

$y^2 = -x^2 + 2y$

$x^2 + y^2 - 2y = 0$

$x^2 + y^2 - 2y + 1 = 1$

$x^2 + (y-1)^2 = 1$

which is a circle centre $(0,1)$ $r = 1$ unit

(d) [1]

(i) $y = \frac{x^2}{4}$

$\frac{dy}{dx} = \frac{2x}{4}$

$= \frac{x}{2}$

At P $\frac{dy}{dx} = \frac{2a}{2}$

$\frac{dy}{dx} = a$ (i)

$m = \tan\beta$ (ii)

$\therefore \tan\beta = a$ (i) = (ii)

(ii) $m_{SP} = \frac{y_2 - y_1}{x_2 - x_1}$ [1]

$= \frac{a^2 - 1}{2a - 0}$

$= \frac{a^2 - 1}{2a}$

$= \frac{1}{2}\left(\frac{a^2}{a} - \frac{1}{a}\right)$

$= \frac{1}{2}\left(a - \frac{1}{a}\right)$

(iii) $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ [2]

$= \frac{a - \frac{1}{2}\left(a - \frac{1}{a}\right)}{1 + \frac{a}{2}\left(a - \frac{1}{a}\right)}$

$= \frac{a - \frac{1}{2}a + \frac{1}{2a}}{1 + \frac{a^2}{2} - \frac{1}{2}}$

$= \frac{\frac{1}{2}a + \frac{1}{2a}}{\frac{1}{2} + \frac{a^2}{2}}$

$= \frac{\frac{a^2+1}{2a}}{\frac{1+a^2}{2}}$

$= \frac{a^2+1}{2a} \times \frac{2}{1+a^2}$

$= \frac{1}{a}$